105 學年 第一學期
離散數學 第一次平時考試
學號： **姓名：Solutions**

1. (25 points) 寫出 $(p\oplus\neg q)\rightarrow(q\wedge r)$ 邏輯公式的真值表。(Write the truth table of logical formula $(p\oplus\neg q)\rightarrow(q\wedge r)$.)

p	q	r	$\neg q$	$p \oplus \neg q$	$q \wedge r$	$(p \oplus \neg q) \rightarrow (q \wedge r)$
T	T	T	F	T	T	**T**
T	T	F	F	T	F	**F**
T	F	T	T	F	F	**T**
T	F	F	T	F	F	**T**
F	T	T	F	F	T	**T**
F	T	F	F	F	F	**T**
F	F	T	T	T	F	**F**
F	F	F	T	T	F	**F**

2. (25 points) 寫出一系列的邏輯相等式以證明 $\neg(p\wedge(p\wedge\neg q))$ 與 $\neg p\vee(p\rightarrow q)$ 是相等的邏輯式，同時寫出每一步驟所使用的相等法則。(Verify $\neg(p\wedge(p\wedge\neg q))$ and $\neg p\vee(p\rightarrow q)$ are logically equivalent by deriving a series of logical equivalences. In every step, write which equivalence is used.)

$\neg(p\wedge(p\wedge\neg q)) \equiv \neg p \vee \neg(p\wedge\neg q))$ (De Morgan's laws for conjunction)
$\equiv \neg p \vee (\neg p \vee \neg\neg q))$ (De Morgan's laws for disjunction)
$\equiv \neg p \vee (\neg p \vee q))$ (Double negation law)
$\equiv \neg p \vee (p \rightarrow q))$ (Logical equivalence of implication)

3. (25 points) 使用真值表證明分配律，及 $p\wedge(q\vee r)$ 和 $(p\wedge q)\vee(p\wedge r)$ 有相同真值表的結果。(Use truth table to verify distributive law, i.e., to show that $p\wedge(q\vee r)$ and $(p\wedge q)\vee(p\wedge r)$ have the same truth value result.)

p	q	r	$q\vee r$	$p\wedge(q\vee r)$	$p\wedge q$	$p\wedge r$	$(p\wedge q)\vee(p\wedge r)$
T	T	T	T	**T**	T	T	**T**
T	T	F	T	**T**	T	F	**T**
T	F	T	T	**T**	F	T	**T**
T	F	F	F	**F**	F	F	**F**
F	T	T	T	**F**	F	F	**F**
F	T	F	T	**F**	F	F	**F**
F	F	T	T	**F**	F	F	**F**
F	F	F	F	**F**	F	F	**F**

Hence, $p\wedge(q\vee r)$ and $(p\wedge q)\vee(p\wedge r)$ have the same truth value result.

4. (25 points) 寫出邏輯式 $\forall x\, \exists y\, (P(x, y) \wedge \exists z\, R(x, y, z))$ 的否定句，並將*否定*的邏輯符號移到最靠近述語 (predicate) 之處
(Write the negation sentence of logical formula $\forall x\, \exists y\, (P(x, y) \wedge \exists z\, R(x, y, z))$ and move the negative connective right before predicates.)

$\neg(\forall x\, \exists y\, (P(x, y) \wedge \exists z\, R(x, y, z))) \equiv \exists x\, \neg(\exists y\, (P(x, y) \wedge \exists z\, R(x, y, z)))$
$\equiv \exists x\, \forall y\, \neg(P(x, y) \wedge \exists z\, R(x, y, z))$
$\equiv \exists x\, \forall y\, \neg P(x, y) \vee \neg\exists z\, R(x, y, z))$
$\equiv \exists x\, \forall y\, \neg P(x, y) \vee \forall z\, \neg R(x, y, z)$