Homework Assignment #1
Solutions

1. Let p and q be the propositions:
   p: I bought a lottery ticket this week.
   q: I won the million dollar jackpot.
   Express each of these propositions as an English sentence.
   (a) $\neg p$ (b) $p \wedge q$ (c) $\neg p \wedge \neg q$ (d) $\neg p \vee (p \wedge q)$
   (a) I did not buy a lottery ticket this week.
   (b) I bought a lottery ticket this week and I won the million dollar jackpot on Friday.
   (c) I did not buy a lottery ticket this week, and I did not win the million dollar jackpot on Friday.
   (d) Either I did not buy a lottery ticket this week, or else I did buy one and won the million dollar jackpot on Friday.

2. Let p, q, and r be propositions:
   p: You get an A on the final exam.
   q: You do every exercise in this book.
   r: You get an A in this class.
   Write these propositions using p, q, and r and logical connectives (including negations):
   (a) You get an A in this class, but you do not do every exercise in this book.
   (b) To get an A in this class, it is necessary for you to get an A on the final.
   (c) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.
   (d) You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.
   (a) $r \wedge \neg q$ (b) $r \rightarrow p$ (c) $(p \wedge q) \rightarrow r$ (d) $r \leftrightarrow (q \vee p)$

In Problems 3 and 4, **1** means **true** (**T**) and **0** means **false** (**F**). You may use **1/0**, **true/false**, or **T/F** in the truth tables.

3. Construct a truth table for each of the following propositional formulas :
   (a) $p \rightarrow \neg p$
   (b) $p \oplus (p \vee q)$
   (c) $(q \rightarrow \neg p) \leftrightarrow (p \leftrightarrow q)$
   (d) $((p \rightarrow q) \rightarrow r) \rightarrow s$
   (a)

p	$\neg p$	$p \rightarrow \neg p$
0	1	1
1	0	0

   (b)

p	q	$p \vee q$	$p \oplus (p \vee q)$
0	0	0	0
0	1	1	1
1	0	1	0
1	1	1	0

(c)

p	q	$\neg p$	$q \to \neg p$	$p \leftrightarrow q$	$(q \to \neg p) \leftrightarrow (p \leftrightarrow q)$
0	0	1	1	1	1
0	1	1	1	0	0
1	0	0	1	0	0
1	1	0	0	1	0

(d)

p	q	r	s	$p \to q$	$(p \to q) \to r$	$((p \to q) \to r) \to s$
0	0	0	0	1	0	1
0	0	0	1	1	0	1
0	0	1	0	1	1	0
0	0	1	1	1	1	1
0	1	0	0	1	0	1
0	1	0	1	1	0	1
0	1	1	0	1	1	0
0	1	1	1	1	1	1
1	0	0	0	0	1	0
1	0	0	1	0	1	1
1	0	1	0	0	1	0
1	0	1	1	0	1	1
1	1	0	0	1	0	1
1	1	0	1	1	0	1
1	1	1	0	1	1	0
1	1	1	1	1	1	1

4. Show that each of these conditional statements is a tautology by using truth tables:
   (a) $[\neg p \wedge (p \vee q)] \to q$
   (b) $[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$

(a)

p	q	$\neg p$	$p \vee q$	$\neg p \wedge (p \vee q)$	$[\neg p \wedge (p \vee q)] \to q$
0	0	1	0	0	1
0	1	1	1	1	1
1	0	0	1	0	1
1	1	0	1	0	1

(b)

p	q	r	$p \vee q$	$p \to r$	$q \to r$	$(p\vee q)\wedge(p\to r)\wedge(q\to r)$	$[(p\vee q)\wedge(p\to r)\wedge(q\to r)]\to r$
0	0	0	0	1	1	0	1
0	0	1	0	1	1	0	1
0	1	0	1	1	0	0	1
0	1	1	1	1	1	1	1
1	0	0	1	0	1	0	1
1	0	1	1	1	1	1	1
1	1	0	1	0	0	0	1
1	1	1	1	1	1	1	1

5. For each of the following propositional formulas and its simplified formulas, show the simplification steps and their reasons: (Do not use truth table.)
   (a) $[(p \vee q) \wedge (p \vee \neg q)] \vee q \Leftrightarrow p \vee q$
   (b) $\neg(p \vee q) \vee [(\neg p \wedge q) \vee \neg q] \Leftrightarrow \neg (q \wedge p)$

(a) $[\underline{(p \vee q) \wedge (p \vee \neg q)}] \vee q$ Distributive Law

$\Leftrightarrow [p \vee \underline{(q \wedge \neg q)}] \vee q$ Negation Law

$\Leftrightarrow [\underline{p \vee \mathit{false}}] \vee q$ Identity Law

$\Leftrightarrow p \vee q$

(b) $\neg(p \vee q) \vee [\underline{(\neg p \wedge q) \vee \neg q}]$ Commutative Law

$\Leftrightarrow \neg(p \vee q) \vee [\underline{\neg q \vee (\neg p \wedge q)}]$ Distributive Law

$\Leftrightarrow \neg(p \vee q) \vee [(\neg q \vee \neg p) \wedge (\neg q \vee q)]$ Negation Law
$\Leftrightarrow \neg(p \vee q) \vee [(\neg q \vee \neg p) \wedge true]$ Identity Law
$\Leftrightarrow \neg(p \vee q) \vee (\neg q \vee \neg p)$ DeMorgan's Law
$\Leftrightarrow \neg(p \vee q) \vee \neg (q \wedge p)$ DeMorgan's Law
$\Leftrightarrow \neg((p \vee q) \wedge (q \wedge p))$ Commutative Law
$\Leftrightarrow \neg((q \wedge p) \wedge (p \vee q))$ Associative Law
$\Leftrightarrow \neg(q \wedge (p \wedge (p \vee q)))$ Absorption Law
$\Leftrightarrow \neg(q \wedge p)$

6. Rewrite each of these statements so that negations appear only within predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives).
   (a) $\neg\exists y\exists x\, P(x, y)$
   (b) $\neg\exists y\, (Q(y) \wedge \forall x\, R(x, y))$
   (c) $\neg\exists y\, (\forall x\exists z\, T(x, y, z) \vee \exists x\forall z\, U(x, y, z))$
   (a) $\neg\exists y\exists x\, P(x, y)$
   $\Leftrightarrow \forall y(\neg\exists x\, P(x, y))$
   $\Leftrightarrow \forall y\forall x\, \neg P(x, y)$

   (b) $\neg\exists y\, (Q(y) \wedge \forall x\, R(x, y))$
   $\Leftrightarrow \forall y\, \neg(Q(y) \wedge \forall x\, R(x, y))$ (DeMorgan's Law)
   $\Leftrightarrow \forall y\, (\neg Q(y) \vee \neg\forall x\, R(x, y))$
   $\Leftrightarrow \forall y\, (\neg Q(y) \vee \exists x\, \neg R(x, y))$

   (c) $\neg\exists y\, (\forall x\exists z\, T(x, y, z) \vee \exists x\forall z\, U(x, y, z))$
   $\Leftrightarrow \forall y\, \neg(\forall x\exists z\, T(x, y, z) \vee \exists x\forall z\, U(x, y, z))$ (DeMorgan's Law)
   $\Leftrightarrow \forall y\, (\neg\forall x\exists z\, T(x, y, z) \wedge \neg\exists x\forall z\, U(x, y, z))$
   $\Leftrightarrow \forall y\, (\exists x\neg\exists z\, T(x, y, z) \wedge \neg\exists x\forall z\, U(x, y, z))$
   $\Leftrightarrow \forall y\, (\exists x\forall z\, \neg T(x, y, z) \wedge \neg\exists x\forall z\, U(x, y, z))$
   $\Leftrightarrow \forall y\, (\exists x\forall z\, \neg T(x, y, z) \wedge \forall x\neg\forall z\, U(x, y, z))$
   $\Leftrightarrow \forall y\, (\exists x\forall z\, \neg T(x, y, z) \wedge \forall x\exists z\, \neg U(x, y, z))$

7. For each of these arguments, translate it into a well-formed formula using predicates, logic connectives, quantifiers. Show the premises and conclusion, prove the conclusion, and explain which rules of inference are used for each step.
   (a) "Linda, a student in this class, owns a red convertible. Everyone who owns a red convertible has gotten at least one speeding ticket. Therefore, someone in this class has gotten a speeding ticket."
   (b) "All movies produced by John Sayles are wonderful. John Sayles produced a movie about coal miners. Therefore, there is a wonderful movie about coal miners."
   (a) Define the following predicates:
   $c(x)$: x is in the class.
   $r(x)$: x owns a red convertible.
   $t(x)$: x has gotten a speeding ticket.
   The given statements can be rewritten as the following logical formula:

$$c(\text{Linda}) \wedge r(\text{Linda}) \wedge (\forall x\, r(x)\rightarrow t(x)) \Rightarrow \exists y\, c(y)\wedge t(y)$$

Step	**Reason**
1. $\forall x\, r(x)\rightarrow t(x)$	Premises

2. $r(\text{Linda})\to t(\text{Linda})$	Step 1 and Rule of Universal Specification
3. $r(\text{Linda})$	Premises
4. $t(\text{Linda})$	Steps 2 and 3 and Modus Ponens
5. $c(\text{Linda})$	Premises
6. $c(\text{Linda})\wedge t(\text{Linda})$	Steps 4 and 5 and Conjunction
7. $\exists y\ c(y)\wedge t(y)$	Step 6 and Rule of Existential Generalization

(b) Define the following predicates:

$s(x)$: x is a movie produced by John Sayles.

$w(x)$: x is a wonderful movie.

$c(x)$: x is a movie about coal miners.

The given statements can be rewritten as the following logical formula:

$$(\forall x\ s(x)\to w(x)) \wedge (\exists x\ s(x)\wedge c(x)) \Rightarrow \exists x\ c(x)\wedge w(x)$$

Step	**Reason**
1. $\exists x\ s(x)\wedge c(x)$	Premises
2. $s(a)\wedge c(a)$	Step 1 and Rule of Existential Specification
3. $s(a)$	Step 2 and Conjunctive Simplification
4. $\forall x\ s(x)\to w(x)$	Premises
5. $s(a)\to w(a)$	Step 3 and Rule of Universal Specification
6. $w(a)$	Steps 4 and 5 and Modus Ponens
7. $c(a)$	Step 2 and Conjunctive Simplification
8. $c(a)\wedge w(a)$	Steps 6 and 7 and Conjunction
9. $\exists x\ c(x)\wedge w(x)$	Step 8 and Rule of Existential Generalization